Algorithms for inverting radio occultation signals in the ionosphere

This document describes the algorithms for inverting ionospheric radio occultation data using the Fortran 77 code `gmrion.f` and related subroutines. The inversion is based on the assumption of local spherical symmetry of the electron density in a large region (a few thousand kilometers in radius) around the ray path tangent points. This assumption may not always be valid, and horizontal ionospheric gradients may significantly affect the retrieved electron density profiles, in particular below the F-layer (sometimes giving large negative or positive electron density). At the same time the geographical location of the ray path tangent points at the top and at the bottom of a profile may differ by several hundred kilometers (horizontal smear). Therefore, retrieved electron density profiles should generally not be interpreted as actual vertical profiles, but rather as a mapping of both vertical and horizontal ionospheric structure into a 1D profile, given a particular occultation geometry.

- Input data (level 1b `ionPhs` file and `parms` file)
  - time, satellite positions, L1 and L2 excess phases
  - calibration mode, processing type (currently only L1−L2 data processing possible), sampling rate, and two altitude range parameters (in `parms5` only)
- Output data (level 2 `ionPrf` file)
  - calibrated total electron content (TEC) and electron density as a function of altitude, as well as many scalar parameters (see below)

Inversion of radio occultation signals in the ionosphere consists of the following steps:

1. Calculation of straight-line impact parameters and corresponding heights, latitudes and longitudes
2. Conversion to Earth-fixed reference frame
3. Re-ordering of impact parameter and phase arrays at both sides of the maximum impact parameter (for TEC calibration)
4. Check for sufficient altitude range
5. Check for time gaps

6. Calibration of phases (depending on calibration mode)
7. Spline interpolation onto a regular grid
8. Calculation of the calibrated occultation TEC
9. Reconstruction of the electron density profile from the calibrated TEC
10. Calculation of the top and bottom altitudes, latitudes, longitudes, and horizontal smear of the tangent point locations
11. Calculation of the F-layer peak density and its altitude, latitude, longitude, and critical frequency
12. Calculation of the vertical TEC between 80 km and the top
13. Estimation of the vertical TEC above the top altitude
14. Calculation of the azimuth angle of the occultation plane

1 Calculation of straight-line impact parameters and corresponding heights, latitudes and longitudes

Bending of GPS signals in the ionosphere is small, and as a first approximation we can assume straight-line propagation. The impact parameter for a given ray is therefore determined by the straight line connecting the GPS transmitter and the LEO receiver. It is calculated as the distance between the tangent point of the straight line and the Earth's center.

An index (taking values -1, 0, or +1) is determined for each pair of satellite positions (each time sample) to indicate if the GPS and LEO satellites geometrically are on opposite sides of the tangent point (-1) or on the same side (+1). This is used later on to identify whether it is a setting or a rising occultation.

The tangent point position is transformed from Cartesian coordinates to latitude, longitude, and height, taking into account the oblateness of the Earth in the determination of geodetic latitude.

Subroutines:

`impact.f` – computes impact parameters and corresponding heights, latitudes, longitudes, and index for identifying occultation flag (rising/setting).

`xyz2g.f` – converts from Earth-centered Cartesian coordinates to latitude, longitude, height coordinates.

2 Conversion to Earth-fixed reference frame

The input satellite orbits are given in the Earth-centered inertial (ECI) frame. Thus the longitude of the tangent points calculated above needs to be transformed to actual geographical longitude with account for rotation of the Earth during an occultation. This depends on the Julian day and the UT (universal time) of the occultation event.

The satellite positions in Cartesian coordinates are subsequently transformed from the ECI frame to the Earth-fixed reference frame.

Subroutines:

`gast.f` – calculates rotation angle between the inertial and the Earth-fixed reference frames for a given date and time (for conversion of longitude).

3 Re-ordering of impact parameter and phase arrays at both sides of the maximum impact parameter (for TEC calibration)

The sample containing the maximum impact parameter is identified. In cases where the collection of data includes samples with positive elevation angle (e.g., GPS/MET), the maximum impact parameter will typically be somewhere in the middle of the impact parameter array. In cases where basically only data with negative elevation angle are collected (e.g., CHAMP), the maximum impact parameter will be at or near the first sample (for a setting occultation). Impact parameter and phase arrays (as well as the time array) are then re-ordered into separate arrays for the negative elevation angle data (occultation side) and for the positive elevation angle data (auxiliary side). Satellite positions and tangent point coordinates are also re-ordered, but only stored on the occultation side. All arrays are finally ordered such that the first sample is the one with the smallest impact parameter.

Subroutines:

`revers.f` – inverts the order of a 1D array.

4 Check for sufficient altitude range

Only occultations with tangent point heights covering the range between 150 km (default value; can be changed in `parms` file) altitude and 1 km (default value; can be changed in `parms` file) below the satellite orbit altitude are processed. If data does not fall within this range, the occultation is discarded. If calibration is performed using the data on the auxiliary side (controlled by input parameter in `parms` file), then it is necessary for the range of impact parameters on the auxiliary side to include the impact parameters on the occultation side, otherwise the occultation is discarded.

Subroutines:

`xyz2g.f` – see above.

5 Check for time gaps

Time gaps may be associated with large cycle slips. The data are checked for time gaps on both the occultation side and the auxiliary side. The identification of a time gap depends on the sampling rate (sampling rate must be correctly specified in the `parms` file). The occultation is discarded if any time gaps are detected.

6 Calibration of phases (depending on calibration mode)

The phase data on the auxiliary side are spline interpolated to the grid of impact parameters on the occultation side. The interpolation is done for the square of the phases as a function of impact parameter, in order to correctly handle the functional form (square root form) of the phase close to the orbit altitude.

The phase data are then calibrated according to the chosen calibration mode (0 or 1 in `parms` file). Calibration mode 1 includes the interpolated auxiliary phase data which are subtracted from the phase data on the occultation side. Mode 1 calibration thus allows estimation of the occultation TEC within the LEO orbit, i.e., excluding the TEC between the GPS satellite altitude and the LEO altitude on the auxiliary side. The assumptions made are that the occultation plane and the LEO plane are near coincident, and that the ionosphere does not change appreciably during the time of data collection. More details about this calibration approach can be found in (Schreiner et al., 1999).

When data on the auxiliary side are not available, one can choose to use calibration mode 0. This calibration approach is somewhat more complicated than mode 1 calibration, and will be described in more detail below. Initially, when mode 0 calibration is chosen, the observed phase for the maximum impact parameter (with tangent point height close to the LEO altitude) is subtracted from the phase data for all samples on the occultation side. Mode 0 calibration is also termed quasi-calibration.

<u>Subroutines:</u>

`splinx.f` – provides natural cubic spline interpolation of a given grid function onto another given grid.

7 Spline interpolation onto a regular grid

A regular height grid with 300 levels is defined to which the orbit coordinates and tangent point coordinates are interpolated.

<u>Subroutines:</u>

`splinx.f` – see above.

8 Calculation of the calibrated occultation TEC

The calibrated occultation TEC (in TECU = 10^{16} electrons/m^2) is calculated from the calibrated phase ΔL as

$$\mathrm{TEC} = \frac{\Delta L}{40.3 \cdot 10^{16}} \frac{f_1^2 f_2^2}{f_1^2 - f_2^2} \ ,$$

where f_1 and f_2 are the GPS L1 and L2 carrier frequencies, respectively. This form is valid when ΔL is related to the L1−L2 phases, which is currently the only processing type available (always chose processing type = 0 in the `parms` input file). In case of mode 0 calibration, the TEC is only initially calibrated (denoted TEC_0 below); final quasi-calibration is done during the reconstruction of the electron density profile as described below.

9 Reconstruction of the electron density profile from the calibrated TEC

For mode 1 calibration, the TEC near the orbit altitude can be written approximately as

$$\mathrm{TEC}(p) \approx 2N_\mathrm{e}(p_\mathrm{max})\sqrt{2p_\mathrm{max}(p_\mathrm{max} - p)} \ ,$$

where p is impact parameter, N_e is electron density, and p_max is the maximum impact parameter assumed to be equal to the orbit radius. The electron density at the orbit altitude is thus found by linear regression of the square of the calibrated TEC for the uppermost few kilometers of tangent point heights. For mode 0 calibration the initially calibrated TEC is about half of the above, assuming that the electron density is spherically symmetrical and falls off exponentially with height above the orbit altitude.

In case of mode 0 calibration, the initially calibrated TEC is further calibrated by adding an analytical estimate of the TEC for the uppermost impact parameter, and subtracting the corresponding analytical estimate of the TEC for positive elevation angles for all samples (thus the quasi-calibration). The analytical estimates are based on the assumption of spherical symmetry, and the assumption that the electron density falls off exponentially with height above the orbit altitude (analytical form pointed out by Sean Healy, ECMWF, presented at the COSMIC Workshop, Boulder, CO, June 2004):

$$\mathrm{TEC}(p) = \mathrm{TEC}_0(p) + N_\mathrm{e}(p_\mathrm{max})\sqrt{\frac{\pi}{2}Hp_\mathrm{max}}\left[1 - \exp\left(\frac{p_\mathrm{max} - p}{H}\right)\mathrm{erfc}\left(\sqrt{\frac{p_\mathrm{max} - p}{H}}\right)\right] \ ,$$

where H is a constant scale height (above the orbit altitude) to be determined, and erfc symbolizes the complementary error function. Initially, H is set to 1000 km, but adjusted iteratively together with $N_\mathrm{e}(p_\mathrm{max})$ when the electron density profile is reconstructed.

The calibrated TEC is spline interpolated to the regular grid of impact parameters on the occultation side, using the square of the TEC as the function variable.

Given the electron density, $N_\mathrm{e}(p_n)$, at the orbit altitude, $p_n = p_\mathrm{max}$, the calibrated TEC is inverted to an electron density profile using the so-called onion peeling method:

$$N_\mathrm{e}(p_i) = \frac{3}{4} \frac{\mathrm{TEC}(p_i)}{\sqrt{2p_i(p_{i+1} - p_i)}} - \sum_{k=1}^{n-i} c_{k,i} N_\mathrm{e}(p_{i+k}) \ ,$$

where the coefficients, $c_{k,i}$, used in `invtec.f` are derived in (Syndergaard et al., 2004).

In case of mode 0 calibration, the above procedure is repeated 10 times, iteratively finding converging estimates of the scale height and electron density at the orbit altitude, and consequently updating the quasi-calibrated TEC and the retrieved electron density profile. In each iteration, the scale height, H, is estimated from the obtained electron density profile by log-linear regression for the uppermost 100 km of tangent point heights.

<u>Subroutines:</u>

`lreg.f` – performs linear regression given a set of function values $y(x)$.

`splinx.f` – see above.

`invtec.f` – performs the inversion of calibrated occultation TEC to electron density profile.

10 Calculation of the top and bottom altitudes, latitudes, longitudes, and horizontal smear of the tangent point locations

The top and bottom altitudes, latitudes, and longitudes are converted to Cartesian coordinates, and the horizontal smear is determined as the distance between the two points projected onto the Earths surface.

<u>Subroutines:</u>

`dist.f` – calculates the horizontal smear distance between two points.

11 Calculation of the F-layer peak density and its altitude, latitude, longitude, and critical frequency

The maximum electron density is obtained by a simple search in the electron density profile above 150 km altitude. The corresponding altitude, latitude, longitude, and critical frequency are stored.

12 Calculation of the vertical TEC between 80 km and the top

The vertical TEC is determined by integration of the electron density between 80 km and the top. Negative values of electron density are not included.

13 Estimation of the vertical TEC above the top altitude

The vertical TEC above the top altitude is estimated by extrapolation using log-linear regression of the electron density profile for the uppermost 100 km of tangent point heights.

Subroutines:

`lreg.f` – see above.

14 Calculation of the azimuth angle of the occultation plane

The azimuth angle of the occultation plane relative to the meridional plane (counted positive eastward) for the top, bottom, and peak density altitudes is calculated as follows. A unit vector normal to the occultation plane (the plane containing the satellites and the Earth's center) is derived from the satellite positions by taking the normalized vector product between the two position vectors, and a unit vector normal to the meridional plane is derived from the tangent point longitude. A unit vector normal to the tangent plane (the plane being tangent to the Earth's surface at the occultation point) is derived from the tangent point latitude and longitude. A unit tangent vector in the occultation plane is derived by taking the vector product between the unit vector normal to the occultation plane and the unit vector normal to the tangent plane. A unit tangent vector in the meridional plane is derived by taking the vector product between the unit vector normal to the meridional plane and the unit vector normal to the tangent plane. Finally, the azimuth angle is determined from the scalar product between the unit tangent vectors in the occultation plane and the meridional plane.

Subroutines:

`azimuth.f` – calculates azimuth of the occultation plane for given positions of GPS and LEO and given latitude and longitude of the ray tangent point.

`vprod.f` – takes vector product between two 3 vectors.

`sprod.f` – takes scalar product between two 3 vectors.

`norm.f` – normalizes a 3 vector.

References

Schreiner, W. S., S. V. Sokolovskiy, C. Rocken, and D. C. Hunt, 1999: Analysis and validation of GPS/MET radio occultation data in the ionosphere. *Radio Sci.*, **34**, 949–966.

Syndergaard, S., E. R. Kursinski, B. M. Herman, E. M. Lane, and D. E. Flittner, 2004: A refractive index mapping operator for variational assimilation of occultation data. *Mon. Weather Rev.*, submitted.